An Effectful Way to Eliminate Addiction to Dependence

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COULD YOU WRITE A HELLO WORLD PROGRAM PLEASE?

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- ... and thus no Hello World!

On Burritos

In less expressive settings, a few workarounds are known.

Typically, on the programming side, use the **monadic** style.

- A type $T : \square \to \square$
- A combinator `return` : $\alpha \to T\,\alpha$
- A combinator `bind` : $T\,\alpha \to (\alpha \to T\,\beta) \to T\,\beta$
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Interpret mechanically effectful programs using this (see Moggi).

This is pervasive in e.g. Haskell.

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Effects are known to implement non-intuitionistic axioms!

- `callcc` $\sim$ classical logic (Griffin '90)
- exceptions $\sim$ Markov's rule (Friedman's trick)
- global monotonous cell $\sim$ $\neg\mathrm{CH}$ (forcing)
- delimited continuations $\sim$ double negation shift
- ...

Achieve this using logical translations, e.g. double-negation.

Alternative Facts

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- And they don't preserve the computational rules of TT

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On the other hand:

- Herbelin showed that CIC + `callcc` is unsound!

In This Talk

1. Adding a vast range of effects to (almost) full TT
    - reader (already done previously with the **forcing translation**)
    - writer, exceptions, non-termination, non-determinism...
    - All with the new **weaning translation**!

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   - No crazy category theory models!
   - So-called **syntactic models**.
   - Compile them on-the-fly into vanilla type theory!

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    - No crazy category theory models!
    - So-called **syntactic models**.
    - Compile them on-the-fly into vanilla type theory!
3. Introducing a generic notion of effectful dependent type theory
    - A simple, sensible restriction of dependent elimination
    - Seemingly compatible with all known effects

Syntactic Models

Define $[\cdot]$ on the syntax and derive the type interpretation $[\![\cdot]\!]$ from it s.t.

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- The correctness of $[\cdot]$ lies in the meta (Darn, Gödel!)
- The translation must preserve typing (Not easy)
- In particular, it must preserve conversion (Argh!)

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Yet, a lot of nice consequences.

- Does not require non-type-theoretical foundations (monism)
- Can be implemented in your favourite proof assistant
- Easy to show (relative) consistency, look at $[\![\texttt{False}]\!]$
- Easier to understand computationally

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- We need to rely on an alternative decomposition (based on CBPV).

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#2. Dependent elimination is hardcore intuitionistic.

- It rules out non-standard inductive terms that exist in CBN + effects
- Reminiscent of Brouwer vs. Bishop mathematics
- Needs to be weakened in presence of effects (« Bishop-style TT »)

My Name is Call, Call-by-Name

TT is intrisically call-by-name because of the conversion rule:

$$\frac{\Gamma \vdash M : B \qquad A \equiv_\beta B}{\Gamma \vdash M : A}$$

where $\equiv_\beta$ is generated by:

$$(\lambda x : A.\, M)\ N \equiv_\beta M\{x := N\}$$

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To be call-by-value, it would require instead $\equiv_{\beta v}$ generated by:

$$(\lambda x : A.\, M)\ V \equiv_{\beta v} M\{x := V\}$$

where V is a value. But that's not TT...

Tell Me Eleinberg-Moore

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For us, a T-algebra will be an inhabitant of:

$$\mathbb{\Box} := \Sigma A : \Box .\ T\ A \to A$$

A few remarks:

- It is hard to formulate the notion of algebra without higher-order types
- We don't require any equations in $\mathbb{\Box}$ (they're quite not algebras)
- It turns out it is not necessary...

Required structure

We assume a monad given by universe-polymorphic terms:

$$\begin{array}{lll} T & : & \square_i \to \square_i \\ \texttt{ret} & : & \Pi(A : \square).\, A \to T\ A \\ \texttt{bind} & : & \Pi(A\ B : \square).\, T\ A \to (A \to T\ B) \to T\ B \end{array}$$

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Furthermore, in Type Theory, types are terms. We want the monad to be **self-algebraic**. This is given by:

$$\begin{array}{lcl} \texttt{El} & : & T\ \square_i \to \square_i \\ \texttt{El}\ (\texttt{ret}\ \square\ M) & \equiv_\beta & M \end{array}$$

A lot of monads appear to be self-algebraic.

The Weaning Translation of the Negative Fragment

$$\begin{array}{lcl}
[x] & := & x \\
[\lambda x : A.\, M] & := & \lambda x : [\![A]\!].\, [M] \\
[M\ N] & := & [M]\ [N] \\
[\square_i] & := & \texttt{ret}\ \boxed{\square}_{i+1}\ (T\ \boxed{\square}_i, \mu_{\square}) \\
[\Pi x : A.\, B] & := & \texttt{ret}\ \boxed{\square}\ (\Pi x : [\![A]\!].\, [\![B]\!], \mu_{\Pi}) \\
[\![A]\!] & := & (\texttt{El}\ [A]).\pi_1 \\
\mu_{\square} & : & T\ (T\ \boxed{\square}) \to \boxed{\square} \\
\mu_{\Pi} & : & T\ (\Pi x : [\![A]\!].\, [\![B]\!]) \to \Pi x : [\![A]\!].\, [\![B]\!]
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[\![A]\!] & := & (\mathtt{El}\ [A]).\pi_1 \\
\mu_\square & : & T\,(T\,\square) \to \square \\
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- Functional fragment untouched, types mangled into algebras
- $[\![\square]\!] \equiv_\beta T\,\square$ and $[\![\Pi x : A.\, B]\!] \equiv_\beta \Pi x : [\![A]\!].\, [\![B]\!]$

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Soundness

If $\Gamma \vdash M : A$ then $[\![\Gamma]\!] \vdash [M] : [\![A]\!]$. (In particular, conversion is preserved.)

Reduction vs. Effects

Nothing fancy in the negative fragment, by the well-known duality.

- Call-by-name: **functions** well-behaved vs. **inductives** ill-behaved
- Call-by-value: **inductives** well-behaved vs. **functions** ill-behaved

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Why is that?

In call-by-name + effects, consider:

$$(\lambda b : \texttt{bool}.\, M)\ \mathbf{fail} \quad \rightsquigarrow \quad \text{non-standard inductive terms}$$

In call-by-value + effects, consider:

$$(\lambda b : \texttt{unit}.\, \mathbf{fail}) \quad \rightsquigarrow \quad \text{invalid } \eta\text{-rule}$$

Weaning Inductive Types

For the sake of explanation, let's focus on a very simple type:

$$\texttt{Inductive bool} := \texttt{true} \mid \texttt{false}.$$

We pose:

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Remark that $[\![\texttt{bool}]\!] \equiv_\beta T\,\texttt{bool}$.

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E-LI-MI-NATE!

We need a bit more structure on T to implement elimination:

$$\begin{array}{lll}\texttt{hbind} & : & \Pi(A:\square)(B:T\,\boxed{\square}).\,T\,A \to (A \to [\![B]\!]) \to [\![B]\!] \\ \texttt{dbind} & : & \Pi(A:\square)(B:A\to T\,\boxed{\square}).\,\Pi(\hat{x}:T\,A). \\ & & \quad (\Pi(x:A).\,[\![B\;x]\!]) \to (\texttt{El}\;(\texttt{hbind}\;A\;[\square]\;\hat{x}\;B)).\pi_1\end{array}$$

subject to:

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Essentially, `hbind` and `dbind` are variants of `bind`.

Remark that the second equation is well-typed iff the first holds.

Interpreting Non-Dependent Elimination

It is easy to provide a non-dependent eliminator using hbind:

$$\begin{array}{lcl} [\texttt{bool\_case}] & : & [\![\Pi P : \square.\, P \to P \to \texttt{bool} \to P]\!] \\ & := & \lambda(P : T\,\boxed{\square})\,(p_t\, p_f : [\![P]\!])\,(\hat{b} : T\,\texttt{bool}). \\ & & \quad \texttt{hbind bool}\ P\ \hat{b}\ (\lambda b.\,\texttt{if}\ b\ \texttt{then}\ p_t\ \texttt{else}\ p_f) \end{array}$$

which has the right reduction rules:

$$\begin{array}{lcl} [\texttt{bool\_case}\ P\ p_t\ p_f\ \texttt{true}] & \equiv_\beta & p_t \\ [\texttt{bool\_case}\ P\ p_t\ p_f\ \texttt{false}] & \equiv_\beta & p_f \end{array}$$

Remember:

$$\begin{array}{l} \texttt{hbind} : \Pi(A : \square)(B : T\,\boxed{\square}).\, T\,A \to (A \to [\![B]\!]) \to [\![B]\!] \\ \texttt{hbind}\ A\ B\ (\texttt{ret}\ A\ M)\ F \equiv_\beta F\ M \end{array}$$

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- Typical of CBN + effects: recall Herbelin's paradox
- Already arose in our forcing translation
- We need to restrict dependent elimination the same way!

Eliminating Addiction to Dependence II

The trick consists in sprinkling a few storage operators. For `bool`:

$$\begin{array}{lcl} [\theta_{\texttt{bool}}] & : & [\![\texttt{bool} \to (\texttt{bool} \to \square) \to \square]\!] \\ & := & [\lambda b.\, \texttt{bool\_case}\ (\texttt{bool} \to \square)\ (\lambda k.\, k\ \texttt{true})\ (\lambda k.\, k\ \texttt{false})\ b] \end{array}$$

- Only defined in the source via non-dependent eliminator
- In particular, agnostic to the actual translation
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Using `dbind`, this allows to implement:

$$[\texttt{bool\_rect}] : [\![\Pi P : \texttt{bool} \to \square.\, P\ \texttt{true} \to P\ \texttt{false} \to \Pi b : \texttt{bool}.\, \theta_{\texttt{bool}}\ b\ P]\!]$$

with the expected reduction rules.

Weaning Everywhere

There are a lot of monads that satisfy the weaning conditions.

- Exception monad $T\ A := A + E$
- Non-determinism $T\ A := A \times \texttt{list}\ A$
- Non-termination $T\ A := \nu X. A + X$
- Writer $T\ A := A \times \texttt{list}\ \Omega$ (the one we need for HELLO WORLD)

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Note that some lead to a logically inconsistent model.

A few monads aren't self-algebraic, e.g. state, reader and continuation.

Logic, at Last

In some inconsistent cases, full dependent elimination is valid.
Most notably, this is the case for the exception monad.

Let's use that to do a Friedman A-translation on steroids!

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Lemmatas

With the exception monad $T\ A := A + E$:

- Full dependent elimination is valid (at the expense of consistency)
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- If A is a first-order type, then $[\![A]\!] \to A + E$.

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Admissibility of Markov's rule in CIC

If A is first-order and $\vdash_{\mathrm{CIC}} \neg\neg A$ then $\vdash_{\mathrm{CIC}} A$.

Moi, j'ai dit linéaire, linéaire ? Comme c'est étrange...

Back to restricted elimination. It turns out we have a semantic criterion for valid dependent predicates.

LINEARITY.

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LINEARITY.

- A concept invented by G. Munch, rephrased recently by P. Levy.
- Little to do with « linear use of variables »
- Essentially, $f : A \to B$ linear in CBN if semantically CBV in A.
- Categorically, f linear iff it is an algebra morphism.
- Storage operators turn freely any morphism into a linear one.
- Can be approximated by a syntactic guard condition.

$$\frac{\Gamma \vdash M : \texttt{bool} \qquad \ldots \qquad P \text{ linear in } b}{\Gamma \vdash \texttt{if } M \texttt{ return } \lambda b.\, P \texttt{ then } N_1 \texttt{ else } N_2 : P\{b := M\}}$$

A Bishop-style Type Theory

We can generalize this restriction to form **Baclofen Type Theory**.

- Subset of CIC
- Independent from the actual translation.
- Works with forcing
- Works with weaning
- Prevents Herbelin's paradox

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BTT is the generic theory to deal with dependent effects
« Bishop-style, effect-agnostic type theory »

(Take that, Brouwerian HoTT!)

Implementation

A nice paper summarizing this talk.

`https://www.pédrot.fr/articles/weaning.pdf`

Just as for the forcing translation we have a Coq plugin for weaning.

`https://github.com/CoqHott/coq-effects`

- Allows to add effects to Coq just today.
- Implement your favourite effectful operators: `fail`, `fix`...
- Compile effectful terms on the fly.
- Allows to reason about them in Coq.

(If time permits, small demo here.)

Conclusion

- A new effectful translation of TT, the weaning translation
  - Cosmic version of Eilenberg-Moore categories
  - Gives both programming and logical features
- An experimentally confirmed notion of effectful type theories, BTT
  - Works for forcing, weaning and CPS
  - Restriction of dependent elimination on linearity guard condition
  - Conjecture: the correct way to add effects to TT
- Implementation of a plugin in Coq
  - Try it out today!

Scribitur ad narrandum, non ad probandum

Thanks for your attention.