The Fire Triangle

How to Mix Substitution, Dependent Elimination and Effects

Pierre-Marie Pédrot, Nicolas Tabareau

Gallinette (INRIA)

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It's Time to CIC Ass and Chew Bubble-Gum

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Yet CIC suffers from a **fundamental** flaw.

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- You want to show the wonders of Coq to a fellow programmer
- You fire your favourite IDE
- … and you're asked the DREADFUL question.

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COULD YOU WRITE A HELLO WORLD?

Sad reality (a.k.a. Curry-Howard)

Intuitionistic Logic $\Leftrightarrow$ **Functional** Programming

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Intuitionistic Logic ⇔ Functional Programming

Coq is even purer than Haskell:

- No mutable state (obviously)
- No exceptions (Haskell has them somehow)
- No arbitrary recursion
- and also no **HELLO WORLD** !

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Thus, the same problem for mathematically inclined users.

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HOW DO I REASON CLASSICALLY?

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Thesis

Non-Intuitionistic Logic $\Leftrightarrow$ **Impure** Programming

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Non-Intuitionistic Logic ⇔ **Impure** Programming

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To program more!

- Non-termination
- Exceptions
- State...

To prove more!

- Classical logic
- Univalence
- Choice...

Something is Rotten in the State of Type Theory

Classical logic does not play well with type theory.

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Effectful theories are always half-broken

- dependent elimination has to be restricted (BTT)
- or consistency forsaken, or worse

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Coincidence? I Think Not!

Definition

A type theory enjoys *substitution* if the following rule is derivable.

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A type theory enjoys *dependent elimination* on booleans if we have:

$$\frac{\Gamma, b : \mathbb{B} \vdash P : \square \qquad \Gamma \vdash \bullet : P\{b := \texttt{true}\} \qquad \Gamma \vdash \bullet : P\{b := \texttt{false}\}}{\Gamma, b : \mathbb{B} \vdash \bullet : P}$$

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Definition

A type theory has *observable effects* if there is a closed term $t : \mathbb{B}$ that is **not observationally equivalent to a value**, i.e. there is a context $C[\cdot]$ s.t.

$$C[\mathtt{true}] \equiv \mathtt{true} \quad \text{and} \quad C[\mathtt{false}] \equiv \mathtt{true} \quad \text{but} \quad C[t] \equiv \mathtt{false}$$

Type Theory on Fire

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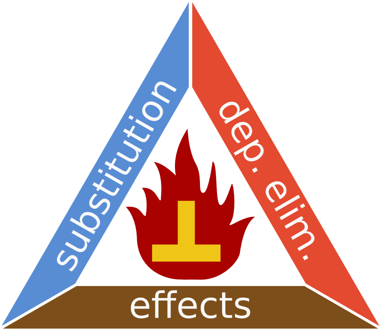

Theorem (Fire Triangle)

substitution + dep. elimination + effects $\vdash$ *logically inconsistent.*

There Is No Free Lunch

The proof is actually straightforward.

Proof.

If C distinguishes boolean values from an effectful term M, prove by dependent elimination $\Pi(b : \mathbb{B}).\ C[b] = \texttt{false}$, apply to M and derive $\texttt{true} = \texttt{false}$. □

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‘But most effects are also observables effects!

So it’s not cheating either.

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And now for a high-level overview of the problem and solutions

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Bad news 1

Typing rules embed the dynamics of programs!

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Bad news 1

Typing rules embed the dynamics of programs!

Bad news 2

Effects make reduction strategies relevant.

Call-by-name vs. Call-by-value

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- Call-by-name: **functions** well-behaved vs. **inductives** ill-behaved
- Call-by-value: **inductives** well-behaved vs. **functions** ill-behaved

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Substitution is a feature of call-by-name

Dependent elimination is a feature of call-by-value

Impossible is not French

Three knobs ⇒ **Four** solutions

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- **Down with effects**: CBN and CBV reconcile

This is good ol' CIC, Keep Calm and Carry on. (†)

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Three knobs $\Rightarrow$ **Four** solutions

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- **Go CBN** and restrict dependent elimination: Baclofen Type Theory

$$\texttt{if } M \texttt{ then } N_1 \texttt{ else } N_2 : \texttt{if } M \texttt{ then } P_1 \texttt{ else } P_2$$

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- **CBV rules**, respect values, and dump substitution

The least conservative approach

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The least conservative approach

- Who cares about consistency? **I want all!**

A paradigm shift: from type theory to dependent languages, e.g. ExTT

Pick Your Side, Comrade

Assuming you want consistent dependent effects...

Call-by-name vs. Call-by-value

Assuming you want consistent dependent effects...

~~**Call-by-n[illegible]-by-value**~~

Call-by-name **and** Call-by-value

CBPV

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∂CBPV

(We had to pick a fancy name, everything else already taken.)

∂CBPV

Justified by all of our syntactic models so far

And we have quite a few!

- Impure Forcing — Unnatural Presheaves
- Reader
- Exceptions — Free algebraic effects
- Self-algebraic monads
- ...

∂CBPV

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- … $\leftarrow$ notice the lack of CPS here

∂CBPV

The main novelties: two for the price of one

- Not one, but **two** parallel hierarchies of universes: $\square_v$ vs. $\square_c$!
- Not one, but **two** let-bindings!

$$\frac{\Gamma \vdash t : F\ A \qquad \color{red}{\Gamma \vdash X : \square_c} \qquad \Gamma, x : A \vdash u : X}{\Gamma \vdash \texttt{let}\ x := t\ \texttt{in}\ u : X}$$

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See the paper for more details

Much More

This was a very high-level talk

Many things I did not discuss here!

- A good notion of purity: thunkability vs. linearity
- Complex ∂CBPV encodings
- Explicit model constructions
- A new look on presheaves

Conclusion

What we did

- Effects and dependent types: you can't have your cake and eat it.
  - $\rightsquigarrow$ Purity, CBN, CBV, Michael Bay?
- Even inconsistent theories have practical interest.
- ∂CBPV a unifying framework for dependent effects

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What we should probably do

- Study more in details CBV type theories
- Try to give a model for classical logic, choice, what else?
- Implement ∂CBPV?