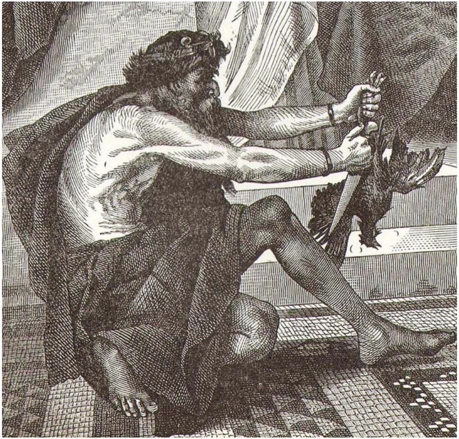
Dans les Boyaux de mon Noyau

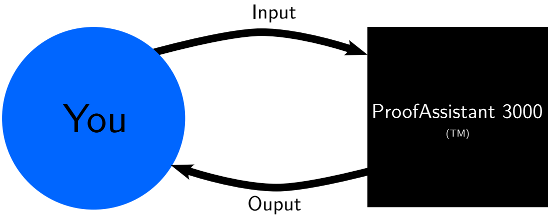


Pierre-Marie Pédrot

(Gallinette, INRIA)

JNIM'24

Des joies† de l'Assistanat à la Preuve



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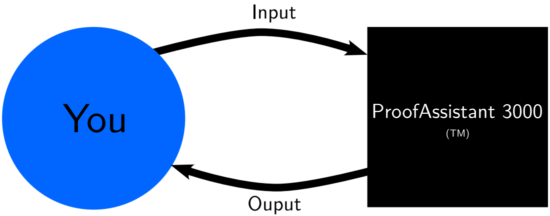

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- Theorem statements and object definitions

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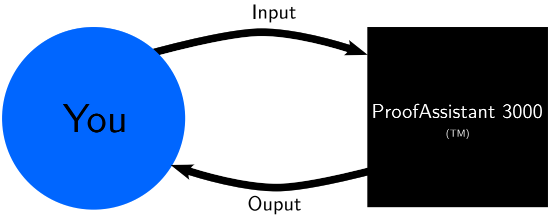

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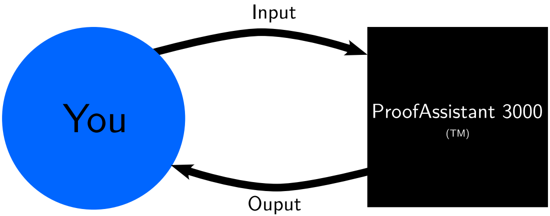

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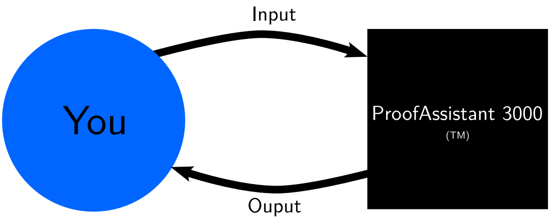

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- (undecypherable dump of some low-level data)

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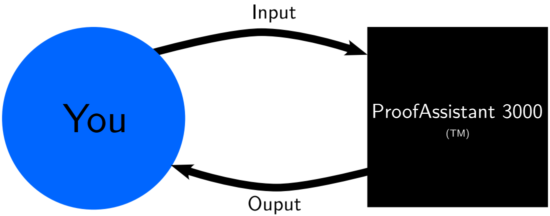

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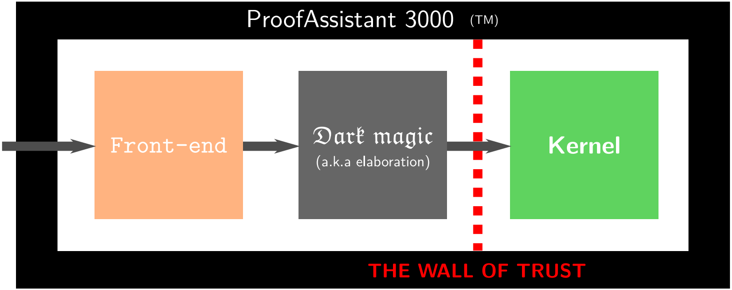
Elle C'est Heffe

A well-known design: The LCF Model



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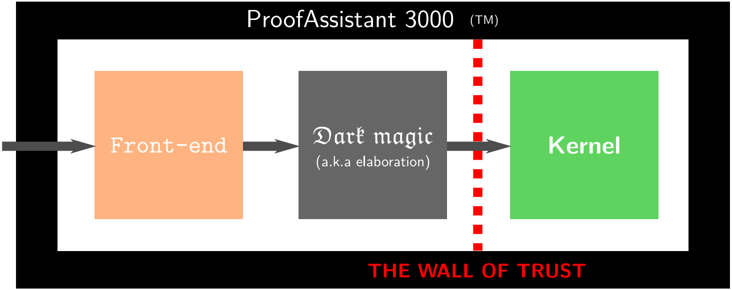

- Clearly delineated Trusted Code Base
- All fancy stuff is outside the TCB
- Soundness is reduced to a *small* hopefully understandable kernel

Two standard kind of kernels in the wild

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First kind: *canal historique* still living in the HOL family

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Noyal-$\lambda\mu$zillac

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Official Position

In this talk, we care about dependent type theories

« Constructions dans un monde qui bouge »

CIC, the calculus of inductive constructions.

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- A powerful dependent type theory
- Programming language or logical foundation?
- The idealized basis of two famous proof assistants

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I am a core Coq developer.

Un théoricien des types c'est un système†

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Thankfully, most of what I am saying should apply to other proof assistants based on dependent type theories.

- Kernel-wise, Coq and Lean are very close.
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Agda

… even if Agda has no separate kernel.

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† — ***Deux c'est une école. Trois, c'est un fork.***

CIC: Programming language or logical foundation?

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Both!

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Level 0: $\vdash (\lambda(x : A).\, x) : A \to A$

- the identity function on A?
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Un Monisme en Deux Minutes

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The pinacle of the Curry-Howard correspondence

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The Coq kernel is *just* a CIC type-checker

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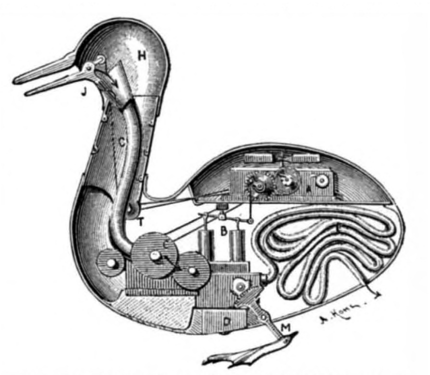

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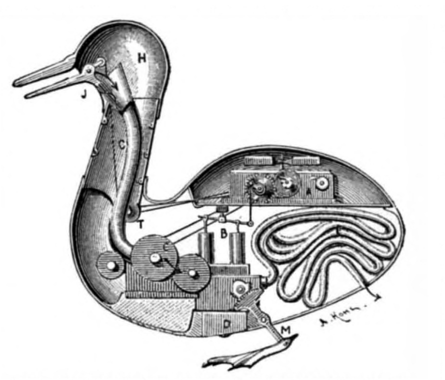

Famous Last Words

"Surely it should be enough to understand CIC to understand the kernel."

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There is no such thing as CIC.

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- A lot of implicit or contradictory stuff
- Folklore / unwritten knowledge

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Our best bet: the MetaCoq project. But that's not today's topic.

Ora Pro Nobis

“The Non-Existent Type Theory is based upon both logic and faith. We have faith that it is implemented; we logically know that it is does not exist because it is not well-defined.”

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We will pretend that CIC exists in the remainder of the talk.

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... for some good notion of CIC.

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Reality has many asperities.

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A specification is not an implementation.

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Consistency: The Logician

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... but I am going to add orthogonal features X, Y and Z.

Rapports de force et personnalités multiples

A Type Theory implementation is a delicate equilibrium.

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- Efficient algorithmics
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The logician, programmer and maintainer are often the same individual.

Pardon My French

— Un noyau, c'est comme une andouillette:
ça doit sentir un peu la merde, mais pas trop.

L'œuf vif du sujet

The setting is now pinned down

In this talk we will discuss three interesting components of the Coq kernel.

Conversion

Universes

Guard

All while keeping the andouillette principle in mind!

Conversion

THE MOST IMPORTANT RULE OF CIC

Le changement, c'est maintenant

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(If you were snoozing away, now is the time to wake up.)

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CONVERSION internalizes computation in the logic

- Not common in usual PL
- Irremediably ties the runtime to the type system
- A landmark of dependent types

Séquent qu'on calcule

$$\Gamma \vdash A \equiv B$$

What is conversion exactly?

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Generated by hardwired basic equations on the language e.g.

- β-reduction: $(\lambda(x : A).\, M)\ N \equiv M\{x := N\}$
- pattern-matching reduction on constructors
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Remember, type-checking should be decidable, so conversion as well.

$\rightsquigarrow$ in particular the kernel must implement conversion.

Liberté, égalité, prouvabilité

Why is conversion critical?

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After all:

- Simple type theories like HOL do not have conversion*
- There are even "weak" dependent type theories without conversion

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- Delegating from the user to the machine is the point of an assistant
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$\rightsquigarrow$ Writing explicitly conversion derivations in CIC is not humanly possible.

Calculo Ergo Sum

Much better: we can take advantage of conversion.

Remember that CIC is a programming language? Let's put this to use.

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Replace logic by computation.

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- prove in CIC that $\Pi(\varphi : \mathtt{formula}).\, \mathtt{check}\ \varphi = \mathtt{true} \to \mathtt{eval}\ \varphi$

To prove $\Phi \in \mathcal{T}$ s.t. $\Phi := \mathtt{eval}\ \varphi$, it is thus enough to **compute** $\mathtt{check}\ \varphi$.

Sum Ergo Calculo

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Morale

Computation matters!

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- Compilation to ZINC-like bytecode / machine code
- The VM is implemented in C, native reuses the OCaml runtime
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Different kind of trade-offs. What is the design space?

(For instance, Lean has an ad-hoc native-like process that only works on closed terms.)

Qui vit par le glaive...

We are not done with conversion yet!

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Let $p, q : \{n : \mathbb{N} \mid \texttt{isEven}\ n\}$.

If $p.1 \equiv q.1$ then we do not have in general $p \equiv q$.

In pen-and-paper proofs one *never ever* cares about that.

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The Dependent Hell

Proofs are programs, and thus relevant.

We would like **more** conversion!

Infère et Damnation

Not just a theoretical issue, this is a real PITA in practice.

Infère et Damnation

Not just a theoretical issue, this is a real PITA in practice.

Sibylline errors in innocuous scripts that require a PhD in type theory to understand.

```
Abstracting over the term "n" leads to a term
fun n0 : nat ⇒ exist (fun n1 : nat ⇒ isEven n1) n0 p = exist (fun n1 : nat ⇒ isEven n1) m q
which is ill−typed.
Reason is: Illegal application:
The term "exist" of type "forall (A : Type) (P : A → Prop) (x : A), P x → {x : A | P x}"
cannot be applied to the terms
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The 4th term has type "isEven n" which should be a subtype of "(fun n : nat ⇒ isEven n) n0".
(cannot unify "isEven n" and "isEven n0")
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(This is just after `rewrite` e where $m, n : \mathbb{N}$ and $e : m = n$.)

Infère et Damnation

Not just a theoretical issue, this is a real PITA in practice.

Sibylline errors in innocuous scripts that require a PhD in type theory to understand.

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A well-known problem that has plagued CIC for years

- Famous hazing for PhD students
- SSReflect even has a design pattern to work around the issue
- Outside of the kernel, not completely satisfactory

La Strictitude, c'est la Stricte Attitude

Recently solved by the introduction of a universe of strict propositions

- After all, proofs are not quite programs
- We don't care about proof contents: "all proofs are born equal."

$$\frac{\Gamma \vdash M : A \qquad \Gamma \vdash N : A \qquad \Gamma \vdash A : \mathtt{SProp}}{\Gamma \vdash M \equiv N : A}$$

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The rules for `SProp` are tricky

- The feature was inspired by foundational work in HoTT
- Required non-trivial changes in the kernel
- Lean notoriously ~~doesn't give a shit~~ is practically-minded

Bien, mais pas top

`SProp` is a game changer

- An elegant solution to perennial issues
- Critical, but not enough

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There are many more limitations to conversion!

$$\mathtt{hd} : \Pi(n : \mathbb{N}).\, \mathtt{vec}\ A\ (1 + n) \rightarrow A \qquad\qquad v : \mathtt{vec}\ A\ (n + 1)$$

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$$\mathtt{hd} : \Pi(n : \mathbb{N}).\, \mathtt{vec}\ A\ (1 + n) \to A \qquad v : \mathtt{vec}\ A\ (n + 1)$$

$\mathtt{hd}\ n\ v$ does not type-check because $1 + n \not\equiv n + 1$

This is much harder to solve.

Retour à l'ATER

Interestingly, these questions were fashionable in the '90s and 00's

- Extensionality in type theory (Hofmann)
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Will the cycle continue?

Universes

Ton Univers Impitoyable

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Standard solution: one has to stratify.

$$\mathtt{Type}_0 : \mathtt{Type}_1 : \mathtt{Type}_2 : \ldots : \mathtt{Type}_n : \mathtt{Type}_{n+1} : \ldots$$

We theoretical computer scientists love natural numbers!

$$(\mathtt{Type}_i)_{i \in \mathbb{N}}$$

- Simple
- Elegant
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Just introduce variables

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The language is simple:

- Variable universe levels i, j...
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(By the way Agda and Lean have a different approach.)

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Three levels ought to be enough for anybody!

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Horror Story

The MetaCoq and QuickChick are (were?) not loadable together.

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I lied (again)

We still need algebraic universe expressions in types.

- In Coq, types are *actually* not terms!
- Some kind of adjunction between types and terms

$$\exists j > i.\, \mathtt{Type}_j \qquad \sim \qquad \mathtt{Type}_{i+1}$$

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The Andouillette Principle

I am not sure I have seen this really publicized anywhere.

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This universe business is currently a hot topic

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Resonates with the multiverse project

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Yet another revival of dormant questions

Guard

Totalitarisme logique

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In paper presentations, for simplicity one uses recursors.

$$\begin{array}{l} \mathtt{rec}_{\mathbb{N}} : P\ \mathsf{O} \rightarrow (\Pi(n : \mathbb{N}).\ P\ n \rightarrow P\ (\mathsf{S}\ n)) \rightarrow \Pi(n : \mathbb{N}).\ P\ n \\ \qquad \begin{array}{lcl} \mathtt{rec}_{\mathbb{N}}\ p_O\ p_S\ \mathsf{O} & \equiv & p_O \\ \mathtt{rec}_{\mathbb{N}}\ p_O\ p_S\ (\mathsf{S}\ n) & \equiv & p_S\ n\ (\mathtt{rec}_{\mathbb{N}}\ n\ p_O\ p_S) \end{array} \end{array}$$

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"$\mathtt{rec}_{\mathbb{N}}$ is universal, because this is the universal property of $\mathbb{N}$."

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Computation matters

Would you conflate a $O(2^n)$ algorithm with $O(1)$ one?

- Intensional behaviour is critical for programming
- Recursors are very bad in call-by-value
- It is not even clear what universality means for conversion
- Whatever this means, recursors are not universal for it

Une idée fixe

Good news: recursors are not fundamental in Coq.

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- Similar to OCaml
- The extracted terms look like what the user wrote
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One can write fixpoints that are not intensionally recursor-encodable.

$$\mathtt{even} : \mathbb{N} \to \mathbb{B}$$

$$\begin{array}{lcl} \mathtt{even}\ \mathsf{O} & := & \mathsf{true} \\ \mathtt{even}\ (\mathsf{S}\ \mathsf{O}) & := & \mathsf{false} \\ \mathtt{even}\ (\mathsf{S}\ (\mathsf{S}\ n)) & := & \mathtt{even}\ n \end{array}$$

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La garde!

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What does the guard enforce?

Minimal service:

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The more expressive the guard, the better.

(Right?)

Terminons-en

The guard condition is probably the least understood kernel component.

- Specification not quite clear, stay tuned
- Organic implementation — *it would be nice if this worked...*
- Decades of tweaks and RFC from users
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I want to give you a foretaste of kern-hell.

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Ingenuous question

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This used to be accepted

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As of Coq 8.19, not accepted, but still morally OK in the abstract.

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The guard condition used to negate propositional extensionality.

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(Mumble something about size issues.)

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This is not a formal specification!

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Who shall guard the guard?

Conclusion

Bad Tripes?

The confidence of the theoretician crashes against the wall of reality

- The real has much more asperities
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Do not be afraid and join us

Scribitur ad narrandum, non ad probandum.

L'absurde ne délivre pas, il lie. Il n'autorise pas tous les actes. Tout est permis ne signifie pas que rien n'est défendu.

Albert Camus.

Thank you for your attention.