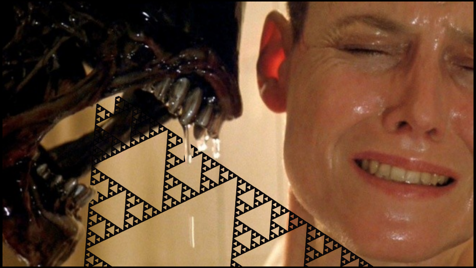
In Cantor Space



No One Can Hear You Stream

Martin Baillon Assia Mahboubi **Pierre-Marie Pédrot**

An I.N.R.I.A. paper

Coming to you
ESOP 2026

Close Encounters of the Dependent Type

— *Mid-twentieth century. Mankind makes a groundbreaking discovery...*

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- Gives computational interpretation to logic
- Sparkles the symbiosis of computer science and logic

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MARTIN-LÖF TYPE THEORY

- Programming and proving united in a single system
- An elegant theory for a more civilized age

Our Cursed Protagonists

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- Parallel worlds of barred truth through Beth semantics

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The Hooded Cult of Topos Theory

- "Embrace local properties"
- Uniting many areas of maths in a single arcane framework
- Glueing eldritch spaces together through **sheaves**

What Lurks in The Dark

Both made the same disturbing findings

Beth semantics $\cong$ **sheaf toposes**

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Sheaves (source: Wikipedia)

Let X be a topological space. A presheaf $F : X \to \mathbf{Set}$ is a sheaf whenever:

1. (Locality) Suppose U is an open set, $\{U_i\}_{i \in I}$ is an open cover of U with $U_i \subseteq U$ for all $i \in I$, and $s, t \in \mathcal{F}(U)$ are sections. If $s|_{U_i} = t|_{U_i}$ for all $i \in I$, then $s = t$.
2. (Glueing) Suppose U is an open set, $\{U_i\}_{i \in I}$ is an open cover of U with $U_i \subseteq U$ for all $i \in I$, and $\{s_i \in \mathcal{F}(U_i)\}_{i \in I}$ is a family of sections. If all pairs of sections agree on the overlap of their domains, that is, if $s_i|_{U_i \cap U_j} = s_j|_{U_i \cap U_j}$ for all $i, j \in I$, then there exists a section $s \in \mathcal{F}(U)$ such that $s|_{U_i} = s_i$ for all $i \in I$.

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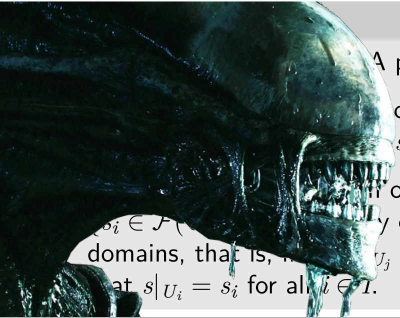

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- Centralizing folklore from different communities in a modern language: **CIC**
- Cleaning historical accidental complexity

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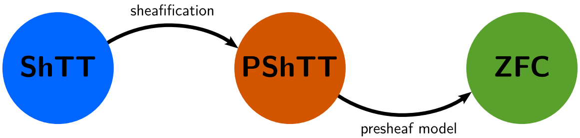
The result: two papers for the price of one!

Part I

A straightforward, typed, computational account of sheaves

A Methodic Incubator

Get rid of the presheaf middle man



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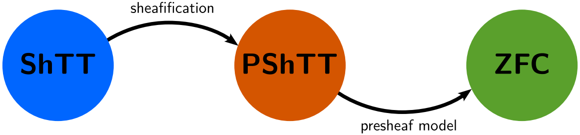

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- Rephrase all the sheaf assembly language internally to **PshTT**

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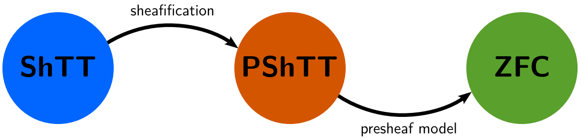

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This one weird trick — classical mathematicians hate him!

The Unmistakable Truth

— Abstractly, our topological space is described as

$$\mathsf{I} : \mathtt{Type} \qquad\qquad \mathsf{O} : \mathsf{I} \to \mathtt{Prop}$$

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— Sheafification is the quotient inductive type

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This gives rise to a straightforward† syntactic model!

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But what on earth is that thing doing computationally?

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Replace unholy objects beyond our computational reach with finite approximations

- A special kind of **interaction trees** baking in continuity
- The ε quotient ensures that such trees behave like functions
- Reminiscent of delimited continuations, game semantics, operating systems...

Part II

A sheafy type theory

A Dash of Man-made Horrors Beyond Your Comprehension

For technical reasons, our syntactic model is flawed

- We need QIT or HITs for $\mathcal{S}$
- We need univalence to implement universes
- Syntactic presheaves do not play well with the latter

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$$\mathbb{N} \rightarrow \mathbb{B}$$

The Cantor Space

i.e. $\mathsf{I} := \mathbb{N}$ and $\mathsf{O}\ (i : \mathsf{I}) := \mathbb{B}$.

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We design $MLTT^{\mathcal{F}}$, an extension of MLTT with a single monster $\alpha : \mathbb{N} \to \mathbb{B}$

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We don't know where the monster is but we can track it:

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What are the metatheoretical properties of MLTT$^{\mathcal{F}}$?

Splitting The Monster Away

We built a logical relation model to justify $\mathrm{MLTT}^{\mathcal{F}}$

- Complete w.r.t. syntax
- Morally the inductive-recursive "realizability with variables" approach
- ... but all logical predicates are sheafy, i.e. up to a tree of splits

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If $\vdash M : \mathbb{N}$ *then there is a split tree* $\mathscr{T}$ *s.t. for each* $(\mathscr{L}, m) \in \mathscr{T}$*,* $M \downarrow_{\mathscr{L}} \overline{m}$*.*

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Full mechanization in Rocq based on `logrel-coq`

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$\text{MLTT}^{\mathcal{F}}$ informs us about MLTT

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Proof. Basically a consequence of the weak canonicity of MLTT$^{\mathcal{F}}$.

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Thus, in Cantor space, no one can hear you stream.

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- Approximating unscrutable higher-order functions by their concrete finite calls

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What lies beyond is unfathomable

- Can we internalize continuity as an operator rather than a rule?
- What kind of topologies can we represent nicely in MLTT?
- Are there other innocuous side-effects out there?

Thanks for your attention.